OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4721

Core Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4721
- List of Formulae (MF1)

Other materials required:
None

Wednesday 18 May 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES
These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

No calculator can be used for this paper

1 Express $3x^2 - 18x + 4$ in the form $p(x+q)^2 + r$. **[4]**

2 **(i)** Sketch the curve $y = \frac{1}{x}$. **[2]**

(ii) Describe fully the single transformation that transforms the curve $y = \frac{1}{x}$ to the curve $y = \frac{1}{x} + 4$. **[2]**

3 Simplify

(i) $\frac{(4x)^2 \times 2x^3}{x}$, **[2]**

(ii) $\left(36x^{-2}\right)^{-\frac{1}{2}}$. **[3]**

4 Solve the simultaneous equations

$$y = 2(x-2)^2, \qquad 3x + y = 26.$$ **[5]**

5 **(i)** Express $\sqrt{300} - \sqrt{48}$ in the form $k\sqrt{3}$, where k is an integer. **[3]**

(ii) Express $\frac{15 + \sqrt{40}}{\sqrt{5}}$ in the form $a\sqrt{5} + b\sqrt{2}$, where a and b are integers. **[3]**

6 Solve the equation $3x^{\frac{1}{2}} - 8x^{\frac{1}{4}} + 4 = 0$. **[5]**

7 Solve the inequalities

(i) $-9 \leqslant 6x + 5 \leqslant 0$, **[3]**

(ii) $6x + 5 < x^2 + 2x - 7$. **[5]**

8 **(i)** Find the coordinates of the stationary point on the curve $y = 3x^2 - \frac{6}{x} - 2$. **[5]**

(ii) Determine whether the stationary point is a maximum point or a minimum point. **[2]**

9 The points $A\,(1, 3)$, $B\,(7, 1)$ and $C\,(-3, -9)$ are joined to form a triangle.

(i) Show that this triangle is right-angled and state whether the right angle is at A, B or C. **[5]**

(ii) The points A, B and C lie on the circumference of a circle. Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. **[7]**

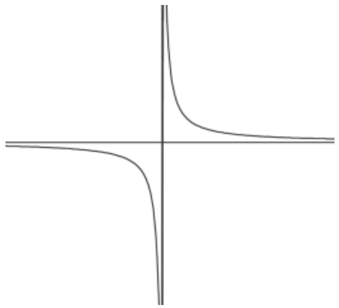
10 A curve has equation $y = (2x - 1)(x + 3)(x - 1)$.

(i) Sketch the curve, indicating the coordinates of all points of intersection with the axes. **[3]**

(ii) Show that the gradient of the curve at the point $P\,(1, 0)$ is 4. **[6]**

(iii) The line l is parallel to the tangent to the curve at the point P. The curve meets l at the point where $x = -2$. Find the equation of l, giving your answer in the form $y = mx + c$. **[4]**

(iv) Determine whether l is a tangent to the curve at the point where $x = -2$. **[3]**

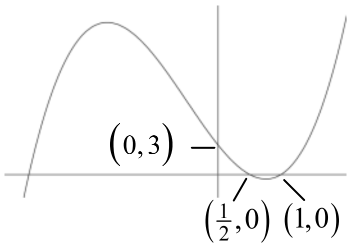
1	$3(x^2-6x)+4$ $=3\left[(x-3)^2-9\right]+4$ $=3(x-3)^2-23$	B1 B1 M1 A1	4 **4**	$p=3$ $(x-3)^2$ seen or $q=-3$ $4-3q^2$ or $\frac{4}{3}-q^2$ (their q) $r=-23$	If p, q, r found correctly, then **ISW** slips in format. $3(x-3)^2+23$ **B1 B1 M0 A0** $3(x-3)-23$ **B1 B1 M1 A1 (BOD)** $3(x-3x)^2-23$ **B1 B0 M1 A0** $3(x^2-3)^2-23$ **B1 B0 M1 A0** $3(x+3)^2-23$ **B1 B0 M1 A1 (BOD)** $3x(x-3)^2-23$ **B0 B1M1A1**
2 (i)		B1 B1	 2	Reasonably correct curve for $y=\frac{1}{x}$ in 1st and 3rd quadrants only Very good curves for $y=\frac{1}{x}$ in 1st and 3rd quadrants **SC** If 0, very good single curve in either 1st or 3rd quadrant and nothing in other three quadrants. **B1**	N.B. Ignore 'feathering' now that answers are scanned. Reasonably correct shape, not touching axes more than twice. Correct shape, not touching axes, asymptotes clearly the axes. Allow slight movement away from asymptote at one end but not more. Not finite.
(ii)	Translation 4 units parallel to y axis	B1 B1	2 **4**	**Must** be translation/translated – not shift, move etc. Or $\begin{pmatrix}0\\4\end{pmatrix}$	For "parallel to the y axis" allow "vertically", "up", "in the (positive) y direction". **Do not accept** "in/on/across/up/along the y axis"
3 (i)	$\frac{16x^2\times 2x^3}{x}$ $=32x^4$	B1 B1	 2	32 x^4	
(ii)	$\frac{1}{6}x$	M1 A1 B1	 3 **5**	6 or $\frac{1}{36^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{36}}$ seen $\frac{1}{6}$ in final answer x (Allow x^1) in final answer	$\frac{1}{\frac{1}{\sqrt{36}}}$ is M0 $\pm\frac{1}{6}$ is **A0**

4	$2x^2 - 8x + 8 = 26 - 3x$	M1		Attempt to eliminate x or y	Must be a clear attempt to reduce to one variable. Condone poor algebra for first mark.
	$2x^2 - 5x - 18 (= 0)$	A1		Correct 3 term quadratic (not necessarily all in one side)	If x eliminated:
	$(2x - 9)(x + 2)(= 0)$	M1		Correct method to solve quadratic	$y = 2(\frac{26-y}{3} - 2)^2$
	$x = \frac{9}{2}, x = -2$	A1		x values correct	Leading to $2y^2 - 89y + 800 = 0$
	$y = \frac{25}{2}, y = 32$	A1	5	y values correct	(2y – 25)(y – 32) =0 etc.
			5	**SR** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**	
5 (i)	$10\sqrt{3} - 4\sqrt{3}$	M1		Attempt to express both surds in terms of $\sqrt{3}$	e.g. $\sqrt{3\text{x}100} - \sqrt{3\text{x}16}$
		B1		One term correct	
	$= 6\sqrt{3}$	A1	3	Fully correct (not $\pm 6\sqrt{3}$)	
(ii)	$\frac{\sqrt{5}(15+\sqrt{40})}{5}$	M1		Multiply numerator and denominator by $\sqrt{5}$ or - $\sqrt{5}$ **or** attempt to express both terms of numerator in terms of $\sqrt{5}$ (e.g. dividing both terms by $\sqrt{5}$)	Check both numerator and denominator have been multiplied
	$= \frac{15\sqrt{5} + 10\sqrt{2}}{5}$	B1		One of a, b correctly obtained	
	$= 3\sqrt{5} + 2\sqrt{2}$	A1	3 **6**	Both a = 3 and b=2 correctly obtained	

6	$k = x^{\frac{1}{4}}$	M1*	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{4}}$	**No marks** unless evidence of substitution (quadratic seen or square rooting or squaring of roots found). = 0 may be implied.
	$3k^2 - 8k + 4 = 0$			
	$(3k-2)(k-2) = 0$	DM1	Correct method to solve a quadratic	Allow $x = x^{\frac{1}{4}}$ as a substitution.
	$k = \frac{2}{3}$ or $k = 2$	A1		**No marks** if straight to quadratic formula to get x = " $\frac{2}{3}$ " x = "2" and no further working
	$x = \left(\frac{2}{3}\right)^4$ or $x = 2^4$	M1	Attempt to calculate k^4	**No marks** if $k = x^{\frac{1}{4}}$ then $3k - 8k^2 + 4 = 0$
	$x = \frac{16}{81}$ or $x = 16$	A1	5 **5**	**SC** If **M0** Spotted solutions **www B1** each Justifies 2 solutions exactly **B3**
	<u>If candidates use $k = x^{\frac{1}{2}}$ and rearrange:</u>			
	$3k - 8\sqrt{k} + 4 = 0$ $8\sqrt{k} = 3k + 4$ $64k = 9k^2 + 24k + 16$	M1*	Substitute, rearrange and square both sides	
	$9k^2 - 40k + 16 = 0$ $(9k-4)(k-4) = 0$	DM1	Correct method to solve quadratic	
	$k = \frac{4}{9}$ or $k = 4$	A1		
	$x = \left(\frac{4}{9}\right)^2$ or $x = 4^2$	M1	Attempt to calculate k^2	
	$x = \frac{16}{81}$ or $x = 16$	A1		
7 (i)	$-14 \le 6x \le -5$	M1	2 equations or inequalities both dealing with all 3 terms resulting in $a \le 6x \le b$, $a \ne -9$, $b \ne 0$	**Do not ISW** after correct answer if contradictory inequality seen.
		A1	-14 and -5 seen **www**	
	$-\frac{7}{3} \le x \le -\frac{5}{6}$	A1 3	Accept as two separate inequalities provided not linked by "or" (must be ≤)	Allow $-\frac{14}{6} \le x \le -\frac{5}{6}$
(ii)	$0 < x^2 - 4x - 12$	M1	Rearrange to collect all terms on one side	**Do not ISW** after correct answer if contradictory inequality seen.
	$(x-6)(x+2)$	M1	Correct method to find roots	
		A1	6, -2 seen	
		M1	Correct method to solve quadratic inequality i.e. $x >$ their higher root, $x <$ their lower root	
	$x > 6$, $x < -2$	A1 5 **8**	(not wrapped, strict inequalities, no 'and')	e.g. for last two marks, $-2 > x > 6$ scores **M1 A0**

8 (i)	$\frac{dy}{dx} = 6x + 6x^{-2}$ $6x + \frac{6}{x^2} = 0$ $x = -1$ $y = 7$	**M1** **A1** **M1** **A1** **A1 ft**	5	Attempt to differentiate (one non-zero term correct) Completely correct Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ Correct value for x - **www** Correct value of y for *their* value of x	**NB** – $x = -1$ (and therefore possibly $y = 7$) can be found from equating the incorrect differential $\frac{dy}{dx} = 6x + 6$ to 0. This could score **M1A0 M1A0A1 ft** If more than one value of x found, allow **A1 ft** for one correct value of y
(ii)	$\frac{d^2y}{dx^2} = 6 - 12x^{-3}$ When $x = -1$, $\frac{d^2y}{dx^2} > 0$ so minimum pt	**M1** **A1 ft**	2 **7**	Correct method e.g. substitutes their x from (i) into their $\frac{d^2y}{dx^2}$ (must involve x) and considers sign. **ft** from their $\frac{dy}{dx}$ differentiated correctly and correct substitution of *their* value of x and consistent final conclusion **NB** If second derivate evaluated, it must be correct (18 for $x = -1$). If more than one value of x used, max **M1 A0**	Allow comparing signs of their $\frac{dy}{dx}$ either side of their "– 1", comparing values of y to their "7" **SC** $\frac{d^2y}{dx^2}$ = a constant correctly obtained from their $\frac{dy}{dx}$ and correct conclusion (ft) **B1**

9 (i)	Gradient of $AB = \frac{1-3}{7-1} = -\frac{1}{3}$ Gradient of $AC = \frac{-9-3}{-3-1} = 3$ Vertex A **OR:** Length of $AB = \sqrt{(7-1)^2+(1-3)^2} = \sqrt{40}$ $AC = \sqrt{(-3-1)^2+(-9-3)^2} = \sqrt{160}$ $BC = \sqrt{(-3-7)^2+(-9-1)^2} = \sqrt{200}$ Shows that $AB^2 + AC^2 = BC^2$ Vertex A	**M1*** **A1** **A1** **M1** **DB1** **M1*** **A1** **A1** **M1** **DB1**	5	Uses $\frac{y_2-y_1}{x_2-x_1}$ for any 2 points One correct gradient (may be for gradient of BC =1) Gradients for both AB and AC found correctly Attempts to show that $m_1 \times m_2 = -1$ oe, accept "negative reciprocal" Correct use of Pythagoras, square rooting not needed Any length or length squared correct All three correct Correct use of Pythagoras to show $AB^2 + AC^2 = BC^2$	Do not allow final mark if vertex A found from wrong working. (Dependent on 1st M 1 A1 A1) Accept BÂC etc for vertex A or "between AB and AC" Allow if marked on diagram. i.e must add squares of shorter two lengths
9 (ii)	Midpoint of BC is $\left(\frac{7+-3}{2}, \frac{1+-9}{2}\right)$ = (2, -4) Length of $BC = \sqrt{(-3-7)^2+(-9-1)^2} = \sqrt{200} = 10\sqrt{2}$ Radius = $5\sqrt{2}$ $(x-2)^2+(y+4)^2 = (5\sqrt{2})^2$ $(x-2)^2+(y+4)^2 = 50$ $x^2+y^2-4x+8y-30=0$	**M1*** **A1** **M1**** **DM1*** **DM1**** **A1** **A1**	7 **12**	Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ o.e. for BC, AB or AC (3 out of 4 subs correct) Correct centre (**cao**) Correct method to find d or r or d^2 or r^2 o.e. for BC, AB or AC (must be consistent with their midpoint if found) $(x-a)^2 + (y-b)^2$ seen for their centre $(x-a)^2 + (y-b)^2$ = their r^2 Correct equation Correct equation in required form	Substitution method 1 (into $x^2 + y^2 + ax + by + c = 0$) Substitutes all 3 points to get 3 equations in a,b,c **M1** At least 2 equations correct **A1** Correct method to find one variable **M1** One of a, b, c correct **A1** Correct method to find other values **M1** All values correct **A1** Correct equation in required form **A1** Alternative markscheme for last 4 marks with f,g,c method: x^2-4x+y^2+8y for their centre **DM1*** $c = (\pm2)^2 + 4^2 - 50$ **DM1**** c = – 30 **A1** Correct equation in required form **A1** Ends of diameter method (p, q) to (c, d): Attempts to use $(x-p)(x-c) + (y-q)(y-d) = 0$ for BC,AC or AB **M2** $(x-7)(x+3) + (y-1)(y+9) = 0$ **A2** for both x brackets correct, **A2** for both y brackets correct $x^2 + y^2 - 4x + 8y - 30 = 0$ **A1** **SC** If **M2 A0 A0** then **B1** if both x brackets correct and **B1** if both y brackets correct for **AC** or **AB**

					<u>Substitution method 2</u>into $(x-p)^2 + (y-q)^2$ = their r^2 Correct method to find d or r or d^2 or r^2 ***M1** Substitutes all 3 points to get 3 equations in p,q **DM1** At least 2 equations correct **A1** Correct method to find one variable **M1** One of p, q correct **A1** Correct equation [$(x-2)^2+(y+4)^2=50$]**A1** Correct equation in required form [$x^2+y^2-4x+8y-30=0$]**A1**
10(i)		**B1** **B1** **B1**	 3	+ve cubic with 3 distinct roots (0, 3) labelled or indicated on y-axis (-3, 0), ($\frac{1}{2}$, 0) and (1, 0) labelled or indicated on x-axis and no other x- intercepts	For first **B1**, left end of curve must finish below x axis and right end must end above x axis. Allow slight wrong curvature at one end but not both ends. No cusp at either turning point. No straight lines drawn with a ruler. Condone (0, 3) as maximum point. To gain second and third **B** marks, there must be an attempt at a curve, not just points on axes. Final **B1** can be awarded for a negative cubic.
(ii)	$2x^2+5x-3$, x^2+2x-3, $2x^2-3x+1$ $(2x^2+5x-3)(x-1)$ $2x^3+3x^2-8x+3$ $\frac{dy}{dx}=6x^2+6x-8$ When $x = 1$, gradient = 4	**B1** **M1** **A1** **M1** **A1** **A1**	 6	Obtain one quadratic factor (can be unsimplified) Attempt to multiply a quadratic by a linear factor Attempt to differentiate (one non-zero term correct) Fully correct expression **www** Confirms gradient = 4 at x = 1 ****AG**	<u>Alternative for first 3 marks:</u> Attempt to expand all 3 brackets with an appropriate number of terms (including an x^3 term) **M1** Expansion with at most 1 incorrect term **A1** Correct, answer (can be unsimplified) **A1** Allow if done in part(i) please check.
(iii)	Gradient of l = 4 On curve, when x = -2, y = 15 $y - 15 = 4(x + 2)$ $y = 4x + 23$	**B1** **B1** **M1** **A1**	 4	May be embedded in equation of line Correct y coordinate Correct equation of line using their values Correct answer **in correct form**	 **M** mark is for any equation of line with any non-zero numerical gradient through (-2, their evaluated y)
(iv)	Attempt to find gradient of curve when x = -2 $6(-2)^2+6(-2)-8=4$ So line is a tangent	**M1** **A1** **A1**	 3 **16**	Substitute x = -2 into their $\frac{dy}{dx}$ Obtain gradient of 4 **CWO** Correct conclusion	<u>Alternatives</u> **1)** Equates equation of l to equation of curve and attempts to divide resulting cubic by $(x + 2)$ **M1** Obtains $(x+2)^2(2x-5)$ (=0) **A1** Concludes repeated root implies tangent at x = -2 **A1** **2)** Equates their gradient function to 4 and uses correct method to solve the resulting quadratic **M1** Obtains $(x+2)(x-1)=0$ oe **A1** Correctly concludes gradient = 4 when x = -2 **A1**